SABES Math Bulletin

Building Research Into Practice
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Welcome to the SABES Math Bulletin

As part of the three-year SABES Math Initiative, quarterly newsletters highlighting research and professional literature related to math instruction will be available to adult educators on line. We attempt to make research accessible to all who teach adult education, but who may not have the time to locate and read full research papers and articles.

Feel free to download and copy the newsletters for friends. It's all part of spreading the word about best practices and general information regarding math teaching and learning.

In This Issue

Issue 3 of the Math Bulletin focuses on *The Components of Numeracy*, a NCSALL occasional paper authored by Lynda Ginsburg, Myrna Manly, and Mary Jane Schmitt. The paper draws upon state, national, and international documents, as it explores the key aspects of numeracy instruction. By understanding the elements that distinguish 'numeracy' from 'mathematics,' adult education teachers gain a fuller sense of the potential for re-shaping their math instruction. Instructional staff may find the ideas useful for group discussion or for review of practices in their own programs.

Inside This Issue

Looking Ahead

In our next issue, due out in June, we will share information and data from *A Review of the Literature in Adult Numeracy: Research and Conceptual Issues* presented by Larry Condelli, Kathy Safford-Ramus, Renee Sherman, Diana Coben, Iddo Gal, and Anestine Hector-Mason for the Office of Vocational and Adult Education and Literacy, U.S. Department of Education and prepared by the American Institutes for Research in Washington, DC. See you then!

Tricia Donovan, Bulletin Editor

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Numeracy Explored

Ed. Note: The following excerpts are compiled from Ginsburg, L., Manly M., and Schmitt, M.J., (2006). *The components of numeracy* (NCSALL Occasional Paper), National Center for the Study of Adult Learning and Literacy, Harvard Graduate School, Cambridge, MA. A complete version of the research may be downloaded at: http://www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf

"Pure mathematics is abstract and context-free, yet 'unlike mathematics, numeracy does not so much lead upward in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life's diverse contexts and situations' (Orril, 2001, p. xviii)." (Ginsburg, et. al., 2006, p. 1)

While some educators argue that adult education programs should emphasize mathematics in all its abstraction, though not necessarily in 'drill and kill' practice, the authors of a recent National Center for the Study of Adult Learning and Literacy (NCSALL) occasional paper, *The Components of Numeracy*, argue for the study of "numeracy." And what is numeracy? According to authors and nationally recognized adult mathematics analysts, Lynda Ginsburg, Myrna Manly, and Mary Jane Schmitt, numeracy "connotes mathematical topics woven into the context of work, community and personal life. … And "unlike pure mathematics, numeracy has a distinctive personal element." (p. 1)

Ginsburg, Manly, and Schmitt analyzed 29 frameworks applicable to adult numeracy. Their review yielded three major components that "form and construct numeracy":

> **"Context** – the use and purpose for which an adult takes on a task with mathematical demands
>
> **Content** – the mathematical knowledge that is necessary for the tasks confronted
>
> **Cognitive and Affective** – the processes that enable an individual to solve problems, and thereby, link the content and context" (p.3)

Context

Context, they maintain, informs frameworks in two ways. It acts as the primary organizing principle in some frameworks and as a secondary principle to mathematics in others. Still a third set of frameworks use mathematics itself as the organizing principle, paying scant explicit attention to context. For each of the three frameworks' types, Ginsburg, Manly and Schmitt offer examples.

Context Continued…

They conclude there is a radical difference between adult and k-12 math frameworks and “that it is the focus on, and prioritization of, context that differentiates an adult numeracy framework from a formal school mathematics framework.” (p.7)

The authors note that math problems are often designed to approximate real situations, but “they generally are structured so that they have only one correct answer,” (p.7) unlike real-life problems which offer a number of variables that must be analyzed before a problem can be solved. Two sample problems provide a neat contrast.

Figure 1: Calculating Batting Averages.

A baseball player’s batting average is the number of hits he gets divided by the number of “at bats.” Joe had 75 “at bats” and made 22 hits. Find his batting average to the nearest thousandth.

1. 0.356
2. 0.333
3. 0.320
4. 0.299
5. 0.293

Figure 2: Which Telephone Plan is best?

Monthly Charge	Plan Features	
$60	900 anytime minutes	$.40 per additional minute Unlimited night and weekend minutes
$80	1400 anytime minutes	$.35 per additional minute Unlimited night and weekend minutes
$200	Unlimited anytime minutes	

According to Ginsburg, Manly, and Schmitt, “There are different judgments as to which contexts are important, the extent to which context is incorporated, and the pedagogical approaches for teaching in or with context. Nonetheless, the overwhelming consensus across the documents we reviewed is this: *context matters*.” (p.10)

Content

While mathematical content varies with time and context, the authors' review of existing frameworks leads them to organize numeracy content around four mathematical strands that resemble closely the National Council of Teachers of Mathematics (NCTM) content strands:

- Number and Operation Sense
- Patterns, Functions, and Algebra
- Measurement and Shape
- Data, Statistics, and Probability

While the descriptive phrases may sound familiar to many readers, the authors make clear that the understanding of these strands has changed. **Number sense** "…considered by most to be arithmetic, has gradually moved from rote, skill-based study to one in which the understanding of concepts is the goal." (p.13) Students are, they say, encouraged to find the relative size of and multiple representations of numbers (1/2, .5, 50%, etc.). In addition, curricula emphasize place value, computation and estimation, the meanings of operations and the relationships between numbers."

"In numeracy, context is the critical factor, so the focus of **algebraic content** for numeracy are those concepts that will help to build the reasoning, skills, and strategies that enable a student to interpret the mathematical demands of situations of the real world." (p.15) Four aspects of algebra are highlighted: language and representation (tables, diagrams and expressions with variables and constants); structures and properties (number and equation properties that allow manipulation of symbols to solve equations or write expressions); mathematical modeling (in which real situations are analyzed for their mathematical relationships and represented using algebraic forms)and functions (which allow analysis of change by studying the relation between quantities).

"The **Measurement and Shape** strand is interwoven with the other strands of numeracy content," (p.17). The researchers identify six concept clusters that current frameworks emphasize in their efforts to foster full participation in today's society: direct measurement (with a tool); indirect measurement (such as using proportionality); angles and lines; attributes and shapes; perimeter, area, and volume, and the coordinate plane. However, they posit that the **data, statistics, and probability** content strand is "the most commonly encountered and relevant of …all" math strands (p.18) in today's world.

Ginsburg, et.al. define data literacy "as the ability of adults to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully." (Idem.) They maintain that a surge in technical applications has resulted in a rush of data from multiple sources. Understanding the flood of data, how it is used and misused, is essential for the numerate adult.

The authors suggest the emergence of three main concept clusters from the frameworks' data that are important to address when developing lessons and curricula for the data, statistics, and probability strand:

1. Collection, organization, and display of data
2. Analysis and interpretation of data
3. Chance

They offer corresponding examples of learning in each of the three concept clusters: (1) "The type of data and the story that it is meant to tell determine the type of chart or graph that is most appropriate for its display;" (2) "changes in the data set can affect the mean and median in different ways, and (3) "In terms of probability, zero represents an impossible event and one represents an event that is certain to occur." (p.19)

Numeracy, they explain, extends far beyond arithmetic. "Numeracy for the twenty-first century is a much richer construct, grounded in the content of these four strands." (Idem.)

Cognitive and Affective

The Components of Numeracy identifies five "*processes that enable an individual to solve problems, and thereby link the context and the content*." (p.21) Of the five processes, three are mentioned in every framework (*) while the others were mentioned in many:

- Conceptual understanding*
- Adaptive reasoning
- Strategic competence*
- Procedural fluency*
- Productive disposition

These processes must be in place in order for a learner to become a numerate adult. The authors describe it this way, saying:

Numerate Adults Must…

(1)have available a rich understanding of the mathematical ideas or concepts involved so as to be able to make sense of the problem. To begin to solve the problem, the person needs to (2) reason or think logically about the relationships within the situation and the concepts that might be related to it. Then, the person needs to (3) formulate the mathematical problem and strategize ways to look at the information, represent it in meaningful ways, and decide, if necessary, how to manipulate numbers to come to a useful solution. Only then does the person (4) perform any needed precise calculations or make estimates, using computational procedures that may require pencil and paper or that may be done mentally or with a calculator. This is an iterative process by which each step must be monitored and reevaluated to see if the process is working as it should, if what is being done continues to seem reasonable, and if changes in direction should be made. Along the way, it is likely that the person will need or want to communicate with others regarding assumptions, strategies, or solutions. This entire process is only possible if the person (5) is emotionally able and willing to engage wit the task, and persevere in the process, dealing with possible confusion, frustration or ambiguity as it arises. (p.21)

Development of cognitive and affective abilities and the inclusion of content and context in curricula are essential to the education of a numerate adult, according to Ginsburg, Manly, and Schmitt.

Looking Forward: The Implications for Adult Education

So what are the implications of the described components of numeracy for adult education practice? The authors suggest:

Four Instructional Emphases

- Include context in curriculum and instruction
- Restructure the scope and sequence of mathematical content (to include all strands at all levels)
- Address all aspects of cognition and affect (create classrooms where problem-solving strategies, mathematical reasoning, positive habits of mind and conceptual understanding receive appropriate attention)
- Create learning environments focused on problem solving

Assessment Changes

"The field should undertake efforts to develop and adopt assessments that are aligned to standards that address the components of adult numeracy," (p.37)

Professional Development Approaches

Professional development should "support teachers who seek to address all the components of numeracy" by developing teachers' mathematical knowledge and their pedagogical skills for addressing cognitive and affective processes.

Together these actions should be undertaken to better serve students. Ginsburg, et.al. conclude that "Envisioning a confident, numerate adult challenges adult educators to develop learners' productive disposition, understanding of concepts, and ability to reason, solve problems, and carryout procedures. This vision of numeracy should revitalize instruction, making learning a more meaningful and lasting experience for adults." (p.41)

Numeracy Components and Subcomponents

Context – the use and purpose for which an adult takes on a task with mathematical demands.

Family or Personal – as a parent, household manager, consumer, financial health-care decision maker, and hobbyist
Workplace—as a worker able to perform tasks on the job and to be prepared to adapt to knew employment demands

Further Learning—as one interested in the more formal aspects of mathematics necessary for further education or training

Community --- as a citizen making interpretations of social situations with mathematical aspects such as the environment, crime, and politics

Content – the mathematical knowledge that is necessary for the tasks confronted

Number and Operation Sense – a sense of how numbers and operations work and how they relate to the world situations that they represent

Patterns, Functions and Algebra – an ability to analyze relationships and change among quantities, generalize and represent them in different ways;, and develop solution methods based on the properties of numbers, operations and equations

Measurement and Shape – knowledge of the attributes of shapes, how to estimate and/or determine the measure of these attributes directly or indirectly, and how to reason spatially

Data, Statistics and Probability – the ability to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully

Cognitive and Affective – the processes that enable an individual to solve problems and, thereby, link the content and the context

Conceptual Understanding – an integrated and functional grasp of mathematical ideas

Adaptive Reasoning – the capacity to think logically about the relationships among concepts and situations

Strategic Competence – the ability to formulate mathematical problems, represent them, and solve them

Procedural Fluency – the ability to perform calculations efficiently and accurately by using paper and pencil procedures, mental mathematics, estimation techniques, and technological aids

Productive Disposition – the beliefs, attitudes, and emotions that contribute to a person's ability and willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects

From Ginsburg, L., Manly, M., and Schmitt, M.J. (2006). *The components of numeracy* (NCSALL occasional Paper). Cambridge, MA: National Center for Study of Adult Literacy and Learning, p.34.. Available: http://www.ncsall.net/resources/research/op_numeracy.pdf

Research Blurbs

U.S. adults performing at numeracy levels 1 and 2 (the lowest of five levels) are about three times more likely to receive social assistance payments from the state (after adjusting for gender, age, education, and income) than those who score in levels 3,4, or 5 (Statistics Canada and OECD, 2005, p. 171) Quoted in *Components of Numeracy*, introduction.) Note: 58.6% of US adults scored in levels 1 and 2 in the Adult Literacy and Lifeskills Survey (ALL).

Eight states have state-wide adult education math standards:
Arizona, Florida, Massachusetts, Ohio, New York, Nevada, Washington, and West Virginia.

According to the analysis performed by the authors of Components of Numeracy (NCSALL, 2006), the adaptive reasoning skills promoted by state-wide adult education standards include the following:

> Determining if results are reasonable (AZ); reasoning to support solutions and ideas, reflecting, connecting, communicating (MA); communicating results (OH); reasoning skills, relationships, connections, communicating mathematical ideas (NY); explain reasoning steps, use and justify different strategies, draw inferences (FL); creative thinking skills (WA).

Further Resources

Adult Education Content Standards Warehouse Web site
http:///ww.adultedcontentstandards.org/howto.asp

SABES Math Initiative Website:
http://www.sabes.org/curriculum/math/index.htm

ALE Wiki: Numeracy Research and Practice
http://wiki.literacytent.org/index.php/Numeracy_Research_and_Practice

NIFL (National Institute for Literacy) Science & Numeracy Special Collections
http://literacynet.org/sciencelincs/

Focus on Basics (Special Math Issue)
www.ncsall.net/index.php?id=156
The September 2000 issue of Focus on Basics (FOB), the quarterly newsletter of the National Center for the Study of Adult Learning and Literacy, is a special focus issue on math. Well worth investigating!

Field Notes Vol. 11, No. 2 (Fall 2001) - Theme: Math
http://www.sabes.org/resources/fieldnotes/vol11/fn112.htm
PDF version: http://www.sabes.org/resources/fieldnotes/vol11/fn112.pdf

Field Notes Vol. 16, No. 1 (Fall/Winter 2006) - Theme: Math
http://www.sabes.org/resources/publications/fieldnotes/vol16/fn161.pdf