SABES Math Bulletin

Building Research Into Practice
Volume 2, Issue 4, June 2008

In This Issue

Mathematics education continues as a priority concern in educational policy circles. In this issue, we share excerpts from the National Mathematics Advisory Panel's report: Foundations for Success. These excerpts share the rationale for the emphasis on mathematics in today's educational system and highlight the need for increased attention to research regarding effective instructional strategies and policies, strengthening students' early math education, and improving teacher preparation. With the issuance of the National Mathematics Advisory Panel's report this spring, we also receive clear guidance regarding the area of major concern in mathematics curriculum: algebra. More detail about this report will be shared in later Bulletin issues.

Also in this issue, we include some "practitioner research" related to algebra. Two Massachusetts ABE instructors: Susan Kahlbaugh and Marilyn Moses, investigated their students' understanding of the equal sign. We share the results here, along with a related math activity.

As we prepare for fall classes, it will be helpful to consider how we teach mathematics to adults to ensure understanding, as well as passing test scores. In particular, it seems, we will be well-advised to contemplate how we teach and how students learn algebra.

Inside

The Equal Sign ========== & How Students Understand It

At the National College Transitions Network Conference in Providence, RI last fall, Mary Jane Schmitt and Tricia Donovan presented the workshop: Algebra for Everyone! As part of that workshop, Donovan shared research about students' misunderstandings of the meaning of the equal sign (operational rather than relational) from:

Knuth, E. J., Stephens, A.C., McNeil, N.M., & Aibali, M.W. (2006) "Does understanding the equal sign matter? Evidence from solving equations," *Journal for Research in Mathematics Education*, NCTM, Vo. 37, No. 4, 297-312.

Knuth, et.al.'s research shared results from middle school students' responses to the following:

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The Equal Sign...

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The following questions are about this statement:

$$3 + 4 = 7$$

(a) The arrow above points to a symbol. What is the name of the symbol?

(b) What does the symbol mean?

(c) Can the symbol mean anything else?

The majority of middle school students responded that the equal sign (=) meant "do something" or "the answer is." They thought of the equal sign as "operational," or indicating the need to perform an operation.

At a SABES Math Initiative meeting, Practitioner Leader Marilyn Moses reported her initial disbelief that adult students thought of the equal sign as a signal for an answer. She was certain they knew "that the equal sign means the values on either side of the sign are equivalent, or the expressions on either side of an equal sign have the same value." She was certain they understood the "relational" meaning of the equal sign – the two sides of an equal sign are related because they are equal in value. However, Moses later reported that she and a colleague asked the Knuth questions (above) of their students and discovered that their adult students shared the misconception of the middle school students. Adult students, too, thought the equal sign meant "total the numbers" or "give the answer." Moses now understood one of her students' barriers to understanding the concepts involved in simplifying equations. Why would you have to add the same number to right as to the left or multiply by the same number on both sides to simplify an equation if the equal sign was a signifier for 'answer' not 'equal value to?'

Moses said she and colleagues now hope to be more conscious of students' perceptions about the equal sign and to be sure they have practice with varying equations, even at the basic levels of math education. Sometimes students will see the answer on the left side of the equation, sometimes they will see equations involving two equivalent expressions, such as 4 + 7 = ___ + 6.

Knuth and fellow researchers also asked students to look at problems such as:

$$5 + 7 = \square + 4$$

Most middle school students filled in the square with the number "12." This affirmed that they understood the operational, not the relational meaning for the equal sign.

Read below to learn what happened when Massachusetts ABE teachers in the Central Region Teacher to Teacher group tried the experiment with their learners:

Dear Tricia,

This note is in relation to our conversation an hour ago about equality. In the third session of T2T the teachers talked about the article, "Children's Understanding of Equality: A Foundation for Algebra". One of the teachers shared that she was surprised by the findings and had tried some problems with her students to see if they did the same thing that the 6th graders did in the article. She teaches in a Pre- GED classroom. All of her students did the same thing that the sixth grade students did:

Example: 5 + 7 = ___ + 4

I asked my ABE students to do five problems of a similar nature and found that 50% answered correctly, while 50% did the same thing as the other teacher's students. It is really important that teachers realize that students may have difficulty with understanding what equality means, and they should take the time to work with them about the meaning in order to ready them for later work in algebra.

Susan Kahlbaugh

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To develop the relational sense of the equal sign, Kahlbaugh and her colleagues now also try to vary the pattern for computation, so that sometimes the answer comes first and so students can practice looking for the missing value in an equation by looking at the relationship between the numbers on both sides.

For instance, in the equation 5 + 7 = ☐ + 4 the four is one less than the five so the missing number must be one more than the seven. The missing number must be '8' in order to make the statement true. Kahlbaugh also uses activities such as "Always, Sometimes, Never True" (see below).

Practitioner Research Basics

Not all researchers are college professors. ABE practitioners can and do conduct research that relates directly to their practice. Basically, teacher-researchers:

- Define a question related to their practice
- Determine a way(s) to collect data that will answer their question
- Collect data
- Analyze data to formulate findings from the research
- Evaluate the data to determine if it was sufficient to answer the question
- Apply findings to practice

For more detailed information about practitioner research, we suggest the links listed below.

For a comprehensive guide to training teachers in practitioner research developed by the National Center for Study of Adult Learning and Literacy (NCSALL), see:

www.ncsall.net/?id=1143

For a guide to practitioner research from the Virginia Adult Educators Research Network, go to:

www.aelweb.vcu.edu/resguide/resguide1.html.

Always, Sometimes, Never True

Are the equations below always, sometimes, or never true? Share your thinking with a partner. Do you both agree? For the same rea -sons?

a) $4 + 5 = 7 + 2$

b) $r2 = r \times r$

c) $3x + 1 = x + 7$

d) $x + 4 = x + 6$

e) $x - 7 = 7 - x$

f) $x \cdot 1/x = 1$

The Final Report of the National Mathematics Advisory Panel

Citation: *National Mathematics Advisory Panel. Foundations for Success: The Final Report of the National Mathematics Advisory Panel, U.S. Department of Education: Washington, DC, 2008.*

Excerpts from the *Foundations for Success: The Final Report of the National Mathematics Advisory Panel Executive Summary*

Background

The eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas. Leading societies have commanded mathematical skills that have brought them advantages in medicine and health, in technology and commerce, in navigation and exploration, in defense and finance, and in the ability to understand past failures and to forecast future developments. History is full of examples.

During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century. This report is about actions that must be taken to strengthen the American people in this central area of learning. …

…We risk our ability to adapt to change. We risk technological surprise to our economic viability and to the foundations of our country's security. National policy must ensure the healthy development of a domestic technical workforce of adequate scale with top-level skills.

But the concerns of national policy relating to mathematics education go far beyond those in our society who will become scientists or engineers. The national workforce of future years will surely have to handle quantitative concepts more fully and more deftly than at present. So will the citizens and policy leaders who deal with the public interest in positions of civic leadership. Sound education in mathematics across the population is a national interest….

…Success in mathematics education also is important for individual citizens because it gives them college and career options, and it increases prospects for future income. A strong grounding in high school mathematics through Algebra II or higher correlates powerfully with access to college, graduation from college, and earning in the top quartile of income from employment. The value of such preparation promises to be even greater in the future. The National Science Board indicates that the growth of jobs in the mathematics-intensive science and engineering workforce is outpacing overall job growth by 3:1. …

On our own "National Report Card"—the National Assessment of Educational Progress (NAEP)—there are positive trends of scores at Grades 4 and 8, which have just reached historic highs. This is a sign of significant progress. Yet other results from NAEP are less positive: 32% of our students are at or above the "proficient" level in Grade 8, but only 23% are proficient at Grade 12. Consistent with these findings is the vast and growing demand for remedial mathematics education among arriving students in four-year colleges and community colleges across the nation.

Moreover, there are large, persistent disparities in mathematics achievement related to race and income—disparities that are not only devastating for individuals and families but also project poorly for the nation's future, given the youthfulness and high growth rates of the largest minority populations. …

Although our students encounter difficulties with many aspects of mathematics, many observers of educational policy see Algebra as a central concern.

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The Final Report...

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The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, where, for more and more students, algebra course work begins. Questions naturally arise about how students can be best prepared for entry into Algebra.

These are questions with consequences, for Algebra is a demonstrable gateway to later achievement. Students need it for any form of higher mathematics later in high school; moreover, research shows that completion of Algebra II correlates significantly with success in college and earnings from employment. In fact, students who complete Algebra II are more than twice as likely to graduate from college compared to students with less mathematical preparation. Among African-American and Hispanic students with mathematics preparation at least through Algebra II, the differences in college graduation rates versus the student population in general are half as large as the differences for students who do not complete Algebra II.

Students who complete algebra 2 are more than twice as likely to graduate from college compared to students with less mathematical preparation.

The essence of the Panel's message is to put first things first. There are six elements, expressed compactly here, but in greater detail later.

- The mathematics curriculum in Grades PreK–8 should be streamlined and should emphasize a well-defined set of the most critical topics in the early grades.
- Use should be made of what is clearly known from rigorous research about how children learn, especially by recognizing a) the advantages for children in having a strong start; b) the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts; and c) that effort, not just inherent talent, counts in mathematical achievement.
- Our citizens and their educational leader-ship should recognize mathematically knowledgeable classroom teachers as having a central role in mathematics education and should encourage rigorously evaluated initiatives for attracting and appropriately preparing prospective teachers, and for evaluating and retaining effective teachers. Instructional practice should be informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers. High-quality research does not support the contention that instruction should be either entirely "student centered" or "teacher directed." Research indicates that some forms of particular instructional practices can have a positive impact under specified conditions.
- NAEP and state assessments should be improved in quality and should carry increased emphasis on the most critical knowledge and skills leading to Algebra.

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- The nation must continue to build capacity for more rigorous research in education so that it can inform policy and practice more effectively.

Positive results can be achieved in a reasonable time at accessible cost, but a consistent, wise, community-wide effort will be required. Education in the United States has many participants in many locales—teachers, students, and parents; state school officers, school board members, superintendents, and principals; curriculum developers, textbook writers, and textbook editors; those who develop assessment tools; those who prepare teachers and help them to continue their development; those who carry out relevant research; association leaders and government officials at the federal, state, and local levels. All carry responsibilities. All can be important to success….

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Math Abbreviations

ABCTE American Board for Certification of Teacher Excellence

ACT American College Testing

CAI Computer-Assisted Instruction

ETS Educational Testing Service

IDA STPI Institute for Defense Analyses Science and Technology Policy Institute

LA Low Achieving

LD Learning Disabilities

NAEP National Assessment of Educational Progress

NCTM National Council of Teachers of Mathematics

NES National Evaluation Systems

SES Socioeconomic Status

STEM Science, Technology, Engineering, and Mathematics

TAI Team Assisted Individualization

TIMSS Trends in International Mathematics and Science Study

PISA Programme for International Student Assessment

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