SABES Math Bulletin

Building Research Into Practice
Volume 3, Issue 4, June 2009

To keep you on your toes as the summer schedule looms before you, the SABES Math Bulletin presents to you an eclectic set of articles in this issue. As promised, we return to look at algebra from different lenses. We share thoughts about arithmetic foundations necessary for algebraic understanding that the National Mathematics Advisory Panel, p.1, included in their Final Report. We reprint an article about questioning techniques used to develop algebraic reasoning by family numeracy researchers Lynda Ginsburg and Kara Jackson (p. 3), and we include a brief look at the history of algebraic notation shared by reader and math teacher, Shana Frank (p.4).

Also included in this issue, we reprint a piece of 'professional wisdom' (p.7) gleaned from a parent-child encounter and described by educator/editor/writer Lenore Balliro, who reminds us all to think more carefully about what we observe students doing and saying in class.

Stepping in a different direction, we share the results of a math literature review on studies examining classroom changes to improve student learning. The studies, according to the authors, Slavin and Lake, indicate that quality professional development for instructors is more important than new technology or textbooks in producing learning gains for math students (p. 5).

Finally, we return to *The Numbers Guy* article to share some math-brain tidbits that support the idea that, as with language, we are hard wired for mathematics.

We hope you peruse the *SABES Math Bulletin* at your leisure, preferably at a beach where the warmth of the sun and the coolness of the water help you digest all the research information we're sharing. Bon voyage!

—Tricia Donovan, Editor

The *Math Bulletin* is a publication of SABES, the System for Adult Basic Education Support, and is funded by the Massachusetts Department of Elementary and Secondary Education. Its mission is to make math research accessible to ABE practitioners.

In This Issue

Critical Foundations of Algebra

National MathematicsAdvisory Panel. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel,* U.S. Department of Education: Washington, DC, 2008. Chapter 4, pp. 15-24 http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

In its recent Final Report, the National Mathematics Advisory Panel outlined three foundational elements in mathematics that students need to master in order to be fully prepared for exploring algebra. Those three critical elements are:

- Fluency with Whole Numbers
- Fluency with Fractions, and
- Particular Aspects of Geometry and Measurement

Fluency, they said, means conceptual understanding and problem-solving skills as well as computational fluency. The Report authors described the three critical foundations in this way:

1. Fluency with Whole Numbers.

By the end of Grade 5 or 6, (students) should have a robust sense of number. This sense of number must include an understanding of place value and the ability to compose and decompose whole numbers. It must clearly include a grasp of the meaning of the basic operations of addition, subtraction, multiplication, and division. It must also include use of the commutative, associative, and distributive properties; computational facility; and the knowledge of how to apply the operations to problem solving. Computational facility requires the automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It also requires fluency with the standard algorithms for addition, subtraction, multiplica-

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tion, and division. Fluent use of the algorithms not only depends on the automatic recall of number facts but also reinforces it. A strong sense of number also includes the ability to estimate the results of computations and thereby to estimate orders of magnitude, e.g., how many people fit into a stadium or how many gallons of water are needed to fill a pool.

2. Fluency with Fractions.

Before they begin algebra course work, middle school (level) students should have a thorough understanding of positive as well as negative fractions. They should be able to locate positive and negative fractions on a number line; represent and compare fractions, decimals, and related percent; and estimate their size. They need to know that sums, differences, products, and quotients (with nonzero denominators) of fractions are fractions, and they need to be able to carry out these operations confidently and efficiently. They should understand why and how (finite) decimal numbers are fractions and know the meaning of percent. They should encounter fractions in problems in the many contexts in which they arise naturally, for example, to describe rates, proportionality, and probability. Beyond computational facility with specific numbers, the subject of fractions, when properly taught, introduces students to the use of symbolic notation and the concept of generality, both being integral parts of algebra.

3. Particular Aspects of Geometry and Measurement.

Middle grade (level) experience with similar triangles is most directly relevant for the study of Algebra: Sound treatments of the slope of a straight line and of linear functions depend logically on the properties of similar triangles. Furthermore, students should be able to analyze the properties of two- and three-dimensional shapes using formulas to determine perimeter, area, volume, and surface. (Final Report, 2008, Chapter 4, pp. 17-18)

The Difference Between Algebra and Arithmetic

"Usually in arithmetic we apply operations to numbers and obtain results after each operation; but in algebra, we usually do not start solving a problem using the given numbers, doing calculations with them, and obtaining a numeric result. In algebra, students have to identify the unknowns, variables and relations among them, and express them symbolically in order to solve the problem." (Martinez, 2007, p.8)

Citation: Martinez, Joseph G. R. (2002). "Building Conceptual Bridges from Arithmetic to Algebra," *Mathematics Teaching in the Middle School,* NCTM, Reston, VA, Vol. 7, No. 6, February 2002, pp. 326-331.

Algebra for All?
How Does Questioning Play a Role?

Reprinted with permission from an Adult Numeracy Network Math Practitioner newsletter article authored by Donna Curry, Spring 2009

In April 2008, the American Educational Research conference was held in New York City. Lynda Ginsburg and Kara Jackson presented their paper, Algebra for All? The Meaning that Mothers Assign to Participation in an Algebra Class. The results of their study with a group of low-income, African-American mothers of elementary-aged children is described in their article in Adults Learning Mathematics – An International Journal, volume 3(2a), November 2008 and can be found at http://www.alm-online.net/images/ALM/journals/almij-volume3_2_a_nov2008.pdf. The following provides a look into the value of questioning in an adult education class.

A group of parents participated in a multi-year project (Parent-Child Numeracy Connections), intended to support a group of parents trying to understand their children's reform-oriented mathematics instruction. The children were part of a project designed to prepare them for college. If the children completed high school, they would receive a scholarship to attend the post-secondary institution of their choice.

While parents first began participating because they were interested in helping their children succeed, they soon developed an interest in learning algebra for themselves, since they had been denied access to algebra when they were in high school. The authors, in analyzing the data, noted that "questioning and making observations were classroom practices that both the instructors and participants engaged in, and that there were particular socio-mathematical norms established around questions and making observations that supported the shifts we were interested in." Questioning was a key factor in moving parents solely interested in learning for their children's sake, to learning for their own sake.

According to the researchers, "Participants developed an understanding of the content at hand because they asked questions and made observations throughout the classes. And, their questions and observations often pushed us as instructors to change the direction we had intended to take during a class period, and instead to engage the learners in sequences of activities that were more challenging than we had initially intended."

Look at a very simple questioning technique that illutrates how students can be pushed in their mathematical thinking. Here is part of an interview between one of the researchers (Lynda) and three of the participants/parents in which they are trying to figure out the difference in y values between two parallel lines they have drawn ($x + 8 = y$ and $x - 10 = y$):

Dionne:	Mine's is 10 from 70.
Lynda:	Yeah, cause you had to subtract 10.
Dionne:	Yeah, this is 10 and this is 8. Then it's a 60.
Lucille:	Wait a minute. 18. 18?
Dionne:	So what was your question again?
Lynda:	How far apart are they?
Lucille:	18? 18 inches?
Lynda:	How do you figure? Why is it 18?
Lucille:	Cause if you were to take and add the difference between that, it would make it 18.
Lynda:	Show me.
Dionne:	From 60 she said.
Lucille:	(Lucille shows with her hand the difference between (70, 60) and (70, 78).) From 60 to 78 would be your 18 inches more. Difference.
Lynda:	Could you know that by looking at your equations? Could you get that 18?
Dionne:	Yes.
Lynda:	How do you figure?
Dionne:	With the plus 8 and the minus 8? 8 plus
Lucille:	Well if you...
Dionne:	10
Lucille:	...wasn't looking at the minus and the plus, it would be 10 and 8 which is 18.

Notice that Lynda asked questions such as "How do you figure? Why is it 18?" Then, when Lucille came up with the answer of 18, Lynda did not simply say, "That's correct." Instead, she further pushed Lucille to explain her reasoning by asking her to "show me." While these questions seem to be so simple, it takes time – and a willingness to reflect on our own teaching –in order to develop the skill of questioning. Lynda illustrates how questioning can push adult learners to new heights in learning.

The Evolution of Algebraic Symbolism

Since the time of the ancient Egyptians and Babylonians, mathematical problems, even situational problems, were completely written out in words as were their solution procedures. This phase of algebraic thinking is usually called the "rhetorical stage." Due to printing and the repeated use of certain terms and words, mathematicians began to use abbreviations to form mathematical relationships. At first, each mathematician or local group of mathematicians had their own system of symbolization but gradually the symbols as well as the procedures became standardized. Below are various examples of how different mathematicians employed symbols to express the modern equation $4x^2 + 3x = 10$."

Nicolas Chuquet	(1484)	4^2 *p*3^1 égualt 10°
Vander Hoecke	(1514)	4 *Se* + 3 Pri dit is ghelijc 10
F. Ghaligai	(1521)	4 ☐ *e* 3c°-10 numeri
Jean Buteo	(1559)	4 ◊ *p*3 *p* [10
Francois Viéte	(c1590)	4*Q* + 3*N* aequatur 10
Thomas Harriot	(1631)	4*aa* + 3*a* ═══ 10
René Descartes	(1637)	4*ZZ* + 3*Z* ơ 10
John Wallis	(1693)	4*XX* + 3*X* = 10

Citation: Swetz, Frank J., Ed. (1994). From five fingers to infinity: A journey through the history of mathematics. Open Court. Chicago.

Wicked Good Math Web Sites

created by Patty Ball, Northeast SABES and posted on www.sabes.org/northeast

1. A-Plus Math - http://www.aplusmath.com/
2. Change Maker - http://www.funbrain.com/cashreg/
3. Consumer Math - http://www.aaamath.com/mny.html
4. Cool Math Sites - http://cte.jhu.edu/techacademy/web/2000/heal/siteslist.htm
5. Everyday Math - http://commtechlab.msu.edu/sites/letsnet/noframes/subjects/math/
6. Instructional Materials for Adult Learners - http://literacy.kent.edu/~illinois/illearners1.htm
7. Interactive Math Games - http://www.cut-the-knot.org/games.shtml
8. Math Archives (lesson plans) - http://archives.math.utk.edu/k12.**html#lessonplans**
9. Math Forum - http://www.mathforum.org/
10. Math Goodies - http://www.mathgoodies.com/
11. Math Links for Teachers - http://www.kent.k12.wa.us/curriculum/math/teachers.html
12. Math Power - http://www.mathpower.com/index.htm
13. Math.com - http://www.math.com/students/references.html

Research Declares: Math Professional Development Pays Off Answering the Question "What Improves Student Learning?"

A recent posting from the Office of Vocational and Adult Education publication *Thursday Notes* exclaimed that "Professional development for math teachers seems to have more of an impact on learning than either new textbooks or technology, according to a research review (published in the 2008 issue of the Review of Educational Research) by the Center for Data-Driven Reform in Education at Johns Hopkins University's School of Education."

The *SABES Math Bulletin* located the original research by Robert E. Slavin and Cynthia Lake and reports on that research below. (The Slavin, Lake citation is posted at the end of this article.)

The professional development Slavin and Lake found to be effective is not 'generic,' nor is it focused solely on improving teachers' content knowledge. "What characterizes the successfully evaluated programs …is a focus on how teachers use instructional process strategies, such as using time effectively, keeping (students) productively engaged, giving (students) opportunities and incentives to help each other learn, and motivating students to be interested in learning mathematics."

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In the overview of their research, Slavin and Lake state: This article reviews research on the achievement outcomes of three types of approaches to improving elementary mathematics: mathematics curricula, computer-assisted instruction (CAI), and instructional process programs.

Study inclusion requirements included use of a randomized or matched control group, a study duration of at least 12 weeks, and achievement measures not inherent to the experimental treatment. Eighty-seven studies met these criteria, of which 36 used random assignment to treatments. There was limited evidence supporting differential effects of various mathematics textbooks.

Effects of CAI were moderate. The strongest positive effects were found for instructional process approaches such as forms of *cooperative learning, classroom management and motivation programs,* and *supplemental tutoring programs* (emphasis ours). The review concludes that programs designed to change daily teaching practices appear to have more promise than those that deal primarily with curriculum or technology alone. (p.427)

Ranked by their effect size the change approaches fell out along these lines:

1. Change classroom practice (math professional development) -- 0.33 effect
2. Supplement curriculum with computer-assisted instruction -- 0.19 effect (mostly on computation)
3. Change the curriculum (textbook) -- 0.10 effect

An effect size of 0.10 is considered statistically significant.

The researchers noted that "With only a few exceptions, effects were similar for disadvantaged and middleclass students and for students of different ethnic backgrounds." (p.476)

Slavin and Lake further categorized the studies as showing "strong evidence of effectiveness" with an effect size of 0.20 or greater and meeting methodology requirements, "moderate evidence of effectiveness," and "limited evidence of effectiveness." They describe the programs that showed strong and moderate effect sizes, saying that the programs represented in each category are "strikingly dif-

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ferent."

In the Strong Evidence category appear five instructional process programs, four of which are cooperative learning programs ….The fifth program is a classroom management and motivation model ….

The Moderate Evidence category is also dominated by instructional process programs, including two supplemental designs, small-group tutoring, and Project SEED, as well as CGI, which focuses on training teachers in mathematical concepts, and CMCD, which focuses on school and classroom management and motivation. Connecting Math Concepts, an instructional process program tied to a specific curriculum, also appears in this category.The only current CAI program with this level of evidence is Classworks. (p.477)

While not discounting the importance of curricula (textbook) choices, the authors state:

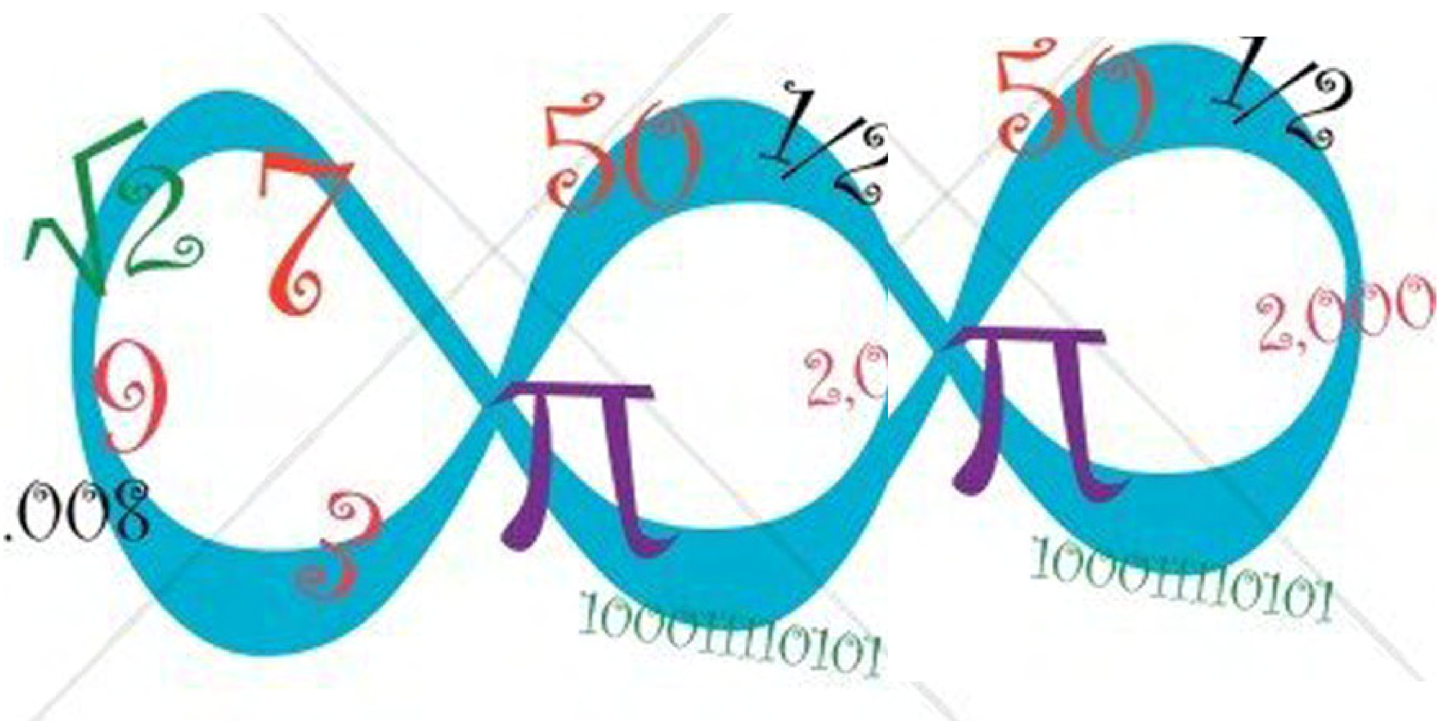
The debate about mathematics reform has focused primarily on curriculum, not on professional development or instruction (see, e.g., American Association for the Advancement of Science, 2000; NRC, 2004). Yet this review suggests that in terms of outcomes on traditional measures, such as standardized tests and state accountability assessments, curriculum differences appear to be less consequential than instructional differences are. (p.482)

The professional development studies that support Slavin and Lake's assertions are typically "extensive" and usually incorporate "follow-up and continuing interactions among the teachers themselves." (p.430)

Slavin and Lake were focusing on improvement strategies in hopes of finding ways to address the achievement gaps between classes and races in the United States.They conclude that in order to address those gaps, it might be helpful to focus on instructional practices, and to "layer" on to those changes the addition of good computer instruction programs to help with computation and making available good textbooks.

Citation:

Slavin, Robert E. & Lake, Cynthia (2008). "Effective Programs in Elementary Mathematics: A Best-Evidence Synthesis," Review of Educational Research 2008, 78, p.427, AERA. DOI: 10.3102/0034654308317473
http://rer.sagepub.com/cgi/content/abstract/78/3/427



What Assumptions Are You Making?
by Lenore Balliro

Rerpinted from the *All Write News,* Adult Literacy Resource Institute, Boston, MA, May 2002

Editor's Note: *In earlier issues of the SABES Math Bulletin, we explained that the definition of 'research' in adult education includes "professional wisdom." The following article was written by one such professional, who used her education background and keen observation skills to examine a key issue in math education – the assumptions that we carry with us into the classroom.*

We make assumptions every minute of the day without thinking about them or making them explicit. We have to. We assume that we'll get up, go to work, and follow a certain routine. We assume that we can turn on the tap and get a glass of water, pick up the phone and call a loved one, turn on our computer and get hooked up to the universe. Of course, there are times (like September 11) when our assumptions are drastically shaken, but on a day-to-day basis we operate from certain givens. If we questioned all of these assumptions all the time we'd go nuts; we couldn't go about our daily business of work or fun.

When we teach we also make assumptions. We assume things about our students and about our teaching. Some of these unexamined assumptions can get in the way of successful teaching and learning. Part of good teaching means identifying our assumptions and questioning them from time to time.

Homework Help

The other day I was reminded of this need to question assumptions about teaching. I was working with my daughter on her second grade math homework. Their math group was studying the concept that you could get to the same answer in different ways. In this case, they were focusing on money. On their worksheets, they were asked to create a total of, say, 50 cents, by using combinations of various coins. So, two quarters, or five dimes, or two nickels and four dimes, or four nickels and three dimes, and so on. The next task asked them to identify different combinations of coins that would equal 75 cents, and so on. You get the picture.

My daughter completed her assignment with ease and speed. (And I was grateful that at this stage in her math schoolwork, I have the math skills to check her work!) But I was curious about what she learned. There wasn't any place on the math homework that described the purpose of the activity. It seemed obvious: you can make a total of X by using many different combinations of coins. But I wanted to hear what she thought. So I asked my daughter,

"What do you think this math homework was trying to teach you?"

She paused for a minute and then said, "They were telling you that you should use the least number of coins you can. So, like, if you are going on the subway, use 2 quarters instead of all those nickels and dimes."

I was surprised at her answer, because I knew she was well aware that you could get to X by a variety or combinations. Starting in kindergarten, they practiced ways to write or illustrate the concept of 100 in as many ways as they possibly could. In first and second grade their investigation of numbers became more sophisticated to expand and reinforce this concept. And now they were applying the same principle to money.

"Oh," I said, "but what if you were saving your quarters for meters and wanted to use your other coins for the subway or bus?"

"I don't know," she said, "I guess that's OK."

"I think the homework was about the idea that you can get to the same total amount by using different combinations of coins," I said.

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“Oh,” she said. “I knew that, but I thought the homework was about something else.”

Now, I don’t know how the teacher framed the homework, or how she analyzed it with the kids afterward. But what this incident showed me is that sometimes we assume that because things seem so obvious to us, the same must be true of our students--that somehow they will make that leap and “get” the learning principle involved without the need for an explicit explanation of the principle. I think that if we neglect to name the principle explicitly, we are losing valuable opportunities to make the connection between something concrete and something abstract.

I remember lots of lost opportunities in my own teaching, and in my work as a staff development facilitator. I know I assigned many creative, engaging activities in class, but I wish I had learned earlier to stop after or before each of the activities and ask: Why do you think we are doing this? How can you apply this in your life (or to your teaching?) While some students will make that connection inductively, others will not; they need the teacher to make the connection between activity and principle clear.

For example, say you are working with students in a math class on ways to solve problems. In one class, you use a pie chart to help you frame a word problem visually. The students are able to solve the problem. You assume that because they solved that problem, they understand that the use of a visual like a pie chart or a graph is a good way to solve a similar problem. Can you always assume that students will make this connection? How can you make it explicit and reinforce it?

How much more powerful each lesson would be if we took a few minutes to check if and how students are “getting” what we are teaching rather than relying on our assumptions that they are understanding things the way we intend. Even if their understanding diverges from our intentions, how much we can learn from listening to their reflections. I wish now I had asked my daughter, for example, why she thought her homework was teaching her to use the least number of coins. It would have revealed something about her thinking. Since I see her all the time, I can always go back and do that. But I can’t go back and fill in the missed opportunities I had to question my assumptions about what students (or teachers) were “getting.”

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Math and the Movies

Extracted from <http://world.std.com/~reinhold/mathmovies.html> June 3, 2009

Proof

Proof is a movie staring Gwyneth Paltrow and Anthony Hopkins adapted from David Auburn’s Tony and Pulitzer winning play. Three of the four main characters are mathematicians. . .Proof’s plot is filed with attempts to prove things: sanity, love, correctness of care decisions, theorems, authorship, adulthood to an older sibling. Even the champaign bottle in the first scene is a mysterious counterexample. . .

Proof’s themes are universal, but the emotional life of mathematicians is dealt with well. Stereotypes are dissected. The math jokes aren’t great but it’s fun to hear the two waves of laughter: from the people who get them immediately and those that have to wait for the playwright’s explanation. Proof’s ending is mathematically satisfying. NYU’s Courant Institute hosted a symposium on Proof.

Die Hard: With A Vengeance (1995)

Bruce Willis and Samuel L. Jackson are given a five gallon jug and a three gallon jug, and must put exactly four gallons of water on a scale to keep a bomb from exploding.

Infinity (1996)

Biopic about Richard Feynman. There’s a priceless scene where he has a calculating duel with a guy with an abacus. Feynman, using pencil and paper, adds a bit slower, but multiplies slightly faster, and really whips him in the cube root competition. Afterward, he explains it all to his fiancee.

Straw Dogs

Dustin Hoffman has moved to his wife’s home town in Cornwall, England in the hope of getting some astrophysics done. His bored wife’s flirtations lead to serious trouble. Somewhere along the line she mischievously changes a plus sign to a minus sign in a set of gravitational equations on a blackboard. Hoffman’s response when he finally notices is by far the best and most realistic portrayal of a mathematician in action in the movies. Caution: The moral of this film is “don’t mess with a mathematician,” so, as you might expect, a great deal of violence occurs.

Numbers Guy Tidbits

In an earlier issue we reported some of the information reported in the *Annals of Science in the New Yorker* (March 3, 2008) regarding the work of researcher Stanis<las Dehaene (the numbers guy). Below we share more various and sundry tidbits gleaned from that article by Jim Holt.

Locating Math in the Brain

In imaging the brain performing various functions, Dehaene and his fellow researchers "found there was a beautiful geometrical organization to the areas that were activated. The eye movements were at the back, the hand movements were in the middle, grasping was in the front, and so on. And right in the middle, we were able to confirm, was an area that cared about number.' The number area lies deep within a fold in the parietal lobe…."

In-born Number Sense

From infancy we seem to have an active sense of number. "Six-month old babies, exposed simultaneously to images of common objects and sequences of drumbeats, consistently gaze longer at the collection of objects that matches the number of drumbeats. By now, it is generally agreed that infants come equipped with a rudimentary ability to perceive and represent number (The same appears to be true for many kinds of animals, including salamanders, pigeons, raccoons, dolphins, parrots, and monkeys." "(W)e have a sense of number that is independent of language, memory and reasoning in general."

The Terror and Errors of Multiplication

"Our number sense endows us with a crude feel for addition, so that even before schooling, children can find recipes for adding numbers. …But multiplication is another matter. It is an 'unnatural practice,' Dehaene is fond of saying, and the reason is that our brains are wired the wrong way. Neither intuition nor counting is of much use, and multiplication facts must be stored in the brain verbally, as strings of words. The list of arithmetical facts to be memorized may be short, but is fiendishly tricky: the same numbers occur over and over, in different orders, with partial overlaps and irrelevant rhymes. (Bilinguals, it has been found, revert to the language they used in school when doing multiplication.) The human memory, unlike that of a computer, has evolved to be associative, which makes it ill-suited to arithmetic, where bits of knowledge must be kept from interfering with one another: if you're trying to retrieve the result of multiplying 7 x 6, the reflex activation of 7 + 6 and 7 x 5 can be disastrous.

So multiplication is a double terror: not only is it remote from our intuitive sense of number; it has to be internalized in a form that clashes with the evolved organization of our memory. The result is that when adults multiply single-digit numbers they make mistakes ten to fifteen per cent of the time. For the hardest problems, like 7 x 8, the error rate can exceed twenty-five percent."

Can you do Division? Divide a loaf by a knife - what's the answer to that? ~Lewis Carroll, *Through the Looking Glass*